

Energy-momentum non-conservation on noncommutative spacetime and the existence of infinite spacetime dimension

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ABSTRACT: From the constructions of the quantum spacetime, a four dimensional quantized spacetime can be embedded in a five dimensional continuous spacetime. Thus to observe from the five dimensional continuous spacetime where the four dimensional quantized spacetime is embedded, there exist the energy-momentum flows between the five dimensional continuous spacetime and the four dimensional quantized spacetime. This makes the energy-momentum not locally conserved generally on the four dimensional quantized spacetime. We propose that energy-momentum tensors of noncommutative field theories constructed from the Noether approach are just the correct forms for the energy-momentum tensors of noncommutative field theories. The non-vanishing of the total divergences of the energy-momentum tensors of noncommutative field theories just reflect that energy-momentum are not locally conserved on noncommutative spacetime. At the same time, from the constructions of the quantum spacetime, we propose that the total spacetime dimension of the quantum spacetime is infinite.

KEYWORDS: Non-Commutative Geometry, Space-Time Symmetries

1 Introduction

People are now still not knowing that whether spacetime is surely discrete and quantized under a very small microscopic scale. However, from certain fundamental principles of physics, it is possible that spacetime may have quantized and noncommutative structures under a very small microscopic scale [1, 2]. This conclusion can also be derived from superstring theories [3, 4]. Field theories on noncommutative spacetime are thus called noncommutative field theories. A lot of research works have been carried out in this area [5, 6].

Noncommutative field theories have many different properties from field theories on commutative spacetime. At the same time, some questions in noncommutative field theories make people feel delusive. One problem among them is about the energy-momentum tensor. It is well known that there are some difficulties to construct a locally conserved energy-momentum tensor for a noncommutative field theory. This problem was first discovered in noncommutative φ^4 scalar field theory [7, 8]. The energy-momentum tensor derived from the Noether approach does not satisfy the conservation equation. Soon after that, the energy-momentum tensors of noncommutative electromagnetic field theory and noncommutative gauge field theory were investigated by several different authors [9, 10, 11]. The same problem has been discovered. The energy-momentum tensors of noncommutative electromagnetic field and Yang-Mills field constructed from the usual approach do not satisfy the conservation equation.

Some modified forms for the energy-momentum tensors of noncommutative field theories have been constructed. In [9], a locally conserved energy-momentum tensor for noncommutative φ^4 scalar field theory was constructed. However, it is not symmetric. In [11], a locally conserved energy-momentum tensor for noncommutative electromagnetic field was constructed. However, it is not symmetric and traceless. A locally conserved non-symmetric energy-momentum tensor has also been constructed in [12] for the renormalizable noncommutative Grosse-Wulkenhaar scalar field theory [13]. On the other hand, with the Wilson functional integral [14, 15], a locally conserved energy-momentum tensor for noncommutative gauge field theory has been constructed in [9]. However, it is not symmetric either. Similarly, an energy-momentum tensor including integrals for noncommutative φ^4 scalar field theory was constructed in [16], although it is symmetric and conserved.

In this paper, we propose a different point of view for the energy-momentum tensor problem of noncommutative field theories. From the constructions of the quantum spacetime, a four dimensional quantized spacetime can be embedded in a five dimensional continuous spacetime. Thus to observe from the five dimensional continuous spacetime where the four dimensional quantized spacetime is embedded, there exist the energy-momentum flows between the five dimensional continuous spacetime and the four dimensional quantized spacetime. This makes the energy-momentum not locally conserved generally on the four dimensional quantized spacetime. We propose that energy-momentum tensors of noncommutative field theories constructed from the Noether approach are just the correct forms for the energy-momentum tensors of noncommutative field theories. The non-vanishing of the total divergences of the energy-momentum tensors of noncommutative field theories just reflect that energy-momentum are not locally conserved on noncommutative spacetime. At the same time, from the constructions of the quantum spacetime, we propose that the total spacetime dimension of the quantum spacetime is infinite.

The content of this paper is organized as follows. In section 2, we analyze the constructions of quantum spacetime and noncommutative field theories. In section 3, from the

constructions of quantum spacetime, we propose that the four dimensional noncommutative spacetime can be embedded in a five dimensional commutative spacetime. Energy-momentum tensors of noncommutative field theories can be obtained from a map relation between field theories on the five dimensional commutative spacetime and field theories on the four dimensional noncommutative spacetime. In section 4, we propose that there exist the energy-momentum flows between the five dimensional continuous spacetime and the four dimensional quantized spacetime. Thus to observe from the four dimensional quantized spacetime, energy-momentum are not locally conserved. We give a derivation for the energy-momentum tensors of noncommutative field theories from the Noether approach parallel to the energy-momentum tensors of corresponding field theories on commutative spacetime. In section 5, from the constructions of the quantum spacetime, we propose that the total spacetime dimension of the quantum spacetime is infinite. In section 6, we give some further discussions.

2 Quantum spacetime and noncommutative field theory

A concrete model of the quantized spacetime was first constructed by Snyder in [1]. In [1], Snyder decomposes the Lorentz rotating generators of the five dimensional Minkowski spacetime into two sets:

$$\mathbf{x}^\mu = ia \left(y^4 \frac{\partial}{\partial y_\mu} - y^\mu \frac{\partial}{\partial y_4} \right), \quad (1)$$

$$\mathbf{M}^{\mu\nu} = -i \left(y^\mu \frac{\partial}{\partial y_\nu} - y^\nu \frac{\partial}{\partial y_\mu} \right), \quad (2)$$

where $\mu, \nu = 0, 1, 2, 3$. As the generators of the $SO(4, 1)$ Lorentz group, they satisfy the commutation relations

$$[\mathbf{x}^\mu, \mathbf{x}^\nu] = ia^2 \mathbf{M}^{\mu\nu}, \quad (3)$$

$$[\mathbf{M}^{\mu\nu}, \mathbf{x}^\lambda] = i(\mathbf{x}^\mu \eta^{\nu\lambda} - \mathbf{x}^\nu \eta^{\mu\lambda}), \quad (4)$$

$$[\mathbf{M}^{\mu\nu}, \mathbf{M}^{\alpha\beta}] = i(\mathbf{M}^{\mu\beta} \eta^{\nu\alpha} - \mathbf{M}^{\mu\alpha} \eta^{\nu\beta} + \mathbf{M}^{\nu\alpha} \eta^{\mu\beta} - \mathbf{M}^{\nu\beta} \eta^{\mu\alpha}). \quad (5)$$

One can see that (5) is just the commutation relations for the generators of the $SO(3, 1)$ Lorentz group, i.e., the Lorentz group of the four dimensional Minkowski spacetime. Equation (3) shows that the coordinates of the four dimensional Minkowski spacetime are quantized. They have the discrete spectra. In fact, because $\mathbf{M}^{\mu\nu}$ are anti-symmetric for the indexes μ and ν , the time coordinate still has the continuous spectrum. Thus Snyder has constructed a quantized spacetime in compatible with the Lorentz invariance with its space coordinates possessing the discrete spectra.

In [2], to combine the principles of general relativity and quantum mechanics, Doplicher *et al.* defined a new algebra for the quantized spacetime which is given by

$$[\mathbf{x}^\mu, \mathbf{x}^\nu] = i\boldsymbol{\theta}^{\mu\nu}, \quad (6)$$

$$[\boldsymbol{\theta}^{\mu\nu}, \mathbf{x}^\lambda] = 0, \quad (7)$$

$$[\boldsymbol{\theta}^{\mu\nu}, \boldsymbol{\theta}^{\alpha\beta}] = 0. \quad (8)$$

Similar relations can also be derived from superstring theories [3, 4]. Carlson *et al.* pointed out that the DFR algebra can be derived from the Snyder algebra through a limit process [17]. To define

$$\mathbf{M}^{\mu\nu} = \frac{1}{b} \boldsymbol{\theta}^{\mu\nu}, \quad (9)$$

then to take the limit

$$a \rightarrow 0, \quad b \rightarrow 0, \quad (10)$$

together with the ratio of a^2 and b being fixed

$$\frac{a^2}{b} \rightarrow 1, \quad (11)$$

the Snyder algebra can be contracted to the DFR algebra. Such a contraction implies that there is a relation between the DFR's quantum spacetime and Snyder's quantum spacetime; although from the group theory language, such a contraction is not standard, as pointed out in [18]. However, to explore the relation between the DFR's quantum spacetime and Snyder's quantum spacetime in detail is not the purpose of this paper.

For noncommutative field theories, they are formulated based on the quantization of spacetime coordinates of (6)–(8). Through the Weyl-Moyal correspondence, every field $\phi(\mathbf{x})$ defined on the noncommutative spacetime is mapped to its Weyl symbol $\phi(x)$ defined on the corresponding commutative spacetime. Meanwhile, the products of field functions are replaced by the Moyal \star -products of their Weyl symbols

$$\phi(\mathbf{x})\psi(\mathbf{x}) \rightarrow \phi(x) \star \psi(x), \quad (12)$$

with the Moyal \star -product defined as

$$\begin{aligned} \phi(x) \star \psi(x) &= e^{\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial \alpha^\mu} \frac{\partial}{\partial \beta^\nu}} \phi(x + \alpha) \psi(x + \beta) \Big|_{\alpha=\beta=0} \\ &= \phi(x) \psi(x) + \sum_{n=1}^{\infty} \left(\frac{i}{2} \right)^n \frac{1}{n!} \theta^{\mu_1 \nu_1} \dots \theta^{\mu_n \nu_n} \partial_{\mu_1} \dots \partial_{\mu_n} \phi(x) \partial_{\nu_1} \dots \partial_{\nu_n} \psi(x); \end{aligned} \quad (13)$$

and the commutators of coordinate operators of (6) are equivalently replaced by the Moyal \star -product commutators of the noncommutative coordinates

$$[x^\mu, x^\nu]_\star = i \theta^{\mu\nu}, \quad (14)$$

where $\theta^{\mu\nu}$ are now the noncommutative parameters.

From the above mentioned Weyl-Moyal correspondence, we acquaint that the Lagrangians of field theories on noncommutative spacetime can be formulated through replacing the products by the Moyal \star -products in the Lagrangians of field theories on commutative spacetime. For example for the Lagrangian of φ^4 scalar field theory on noncommutative spacetime we have

$$\mathcal{L} = \frac{1}{2} \partial^\mu \varphi(x) \star \partial_\mu \varphi(x) - \frac{1}{2} m^2 \varphi(x) \star \varphi(x) - \frac{1}{4!} \lambda \varphi(x) \star \varphi(x) \star \varphi(x) \star \varphi(x). \quad (15)$$

Correspondingly, for electromagnetic field on noncommutative spacetime, its Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}(x) \star F_{\mu\nu}(x), \quad (16)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - i[A_\mu(x) \star A_\nu(x) - A_\nu(x) \star A_\mu(x)] . \quad (17)$$

The last term in the definition of $F_{\mu\nu}$ is in order to make the noncommutative $U(1)$ gauge field theory be compatible with the noncommutative $U(N)$ gauge field theories. Because in (15)–(17), the coordinates x^μ satisfy the commutation relations (14), these noncommutative field theories are defined on the four dimensional quantized spacetime \mathbf{x}^μ .

3 Energy-momentum tensor on noncommutative spacetime through the map relation

On the other hand, from the operator construction of (1)–(5), there exists a five dimensional commutative spacetime y^μ , $\mu = 0, 1, \dots, 4$. From the above constructions of the four dimensional quantized spacetime \mathbf{x}^μ , its points take the eigenvalues of the operators (1). Thus its point set can be regarded as a subset of the five dimensional continuous spacetime y^μ . This means that the four dimensional quantized spacetime \mathbf{x}^μ can be embedded in the five dimensional continuous spacetime y^μ through a certain manner. Its point set composes a four dimensional hyper-surface of the five dimensional continuous spacetime. Or we can regard that the four dimensional quantized spacetime \mathbf{x}^μ is a subspace of the five dimensional commutative spacetime y^μ . Therefore there exist a map relation between the five dimensional commutative spacetime y^μ and the four dimensional quantized spacetime \mathbf{x}^μ .

In the five dimensional commutative spacetime y^μ , there exist the commutative field theories. For example for the φ^4 scalar field theory, its Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} \partial^\mu \varphi(y) \partial_\mu \varphi(y) - \frac{1}{2} m^2 \varphi^2(y) - \frac{1}{4!} \lambda \varphi^4(y) . \quad (18)$$

For the electromagnetic field theory, its Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}(y) F_{\mu\nu}(y) , \quad (19)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu(y) - \partial_\nu A_\mu(y) . \quad (20)$$

These field theories are defined in the five dimensional commutative spacetime y^μ . From the five dimensional commutative spacetime field theories to the four dimensional noncommutative spacetime field theories, one only need to replace the product by the Moyal \star -product in the Lagrangians. Thus there is also a map relation between the five dimensional commutative field theories and the four dimensional noncommutative field theories. However, we conjecture that such a map relation not only exists in the Lagrangians between these two kinds of field theories, but also exists in some other expressions between these two kinds of field theories. In fact, such a map relation also exists in the field equations. Thus, it is reasonable to conjecture that such a map relation also exists in the expressions of the energy-momentum tensors of these two kinds of field theories.

For field theories on the five dimensional commutative spacetime y^μ , their energy-momentum tensors can be obtained through the Noether approach. For the φ^4 scalar field theory, we have

$$\mathcal{T}_{\mu\nu} = \partial_\mu \varphi(y) \partial_\nu \varphi(y) - g_{\mu\nu} \mathcal{L}(y) . \quad (21)$$

It is symmetric and locally conserved. For the electromagnetic field, through the canonical approach and the Belinfante procedure, its energy-momentum tensor can be constructed as

$$\mathcal{T}_{\mu\nu} = \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}(y)F_{\alpha\beta}(y) - F_{\mu\lambda}(y)F_{\nu}^{\lambda}(y) . \quad (22)$$

It is symmetric, traceless, locally conserved, and also gauge invariant. From the map relation between the five dimensional commutative field theories and four dimensional noncommutative field theories, we can write down the energy-momentum tensors of the corresponding field theories on four dimensional noncommutative spacetime. For φ^4 scalar field theory we have

$$\mathcal{T}_{\mu\nu} = \frac{1}{2}(\partial_{\mu}\varphi(x) \star \partial_{\nu}\varphi(x) + \partial_{\nu}\varphi(x) \star \partial_{\mu}\varphi(x)) - g_{\mu\nu}\mathcal{L}(x) . \quad (23)$$

In (23), an additional symmetrizing procedure has been made in order to make the expression symmetric for the reason that Moyal \star -product is not symmetric between two different functions. For the electromagnetic field, we have

$$\mathcal{T}_{\mu\nu} = \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}(x) \star F_{\alpha\beta}(x) - \frac{1}{2}(F_{\mu\lambda}(x) \star F_{\nu}^{\lambda}(x) + F_{\nu\lambda}(x) \star F_{\mu}^{\lambda}(x)) . \quad (24)$$

The same reason as above, an additional symmetrizing procedure has been made. Such an energy-momentum tensor is symmetric and traceless [10]. Under the gauge transformation $\delta A_{\mu} = D_{\mu}\Omega$, it transforms covariantly

$$\mathcal{T}_{\mu\nu} \rightarrow \Omega \star \mathcal{T}_{\mu\nu} \star \Omega^{-1} . \quad (25)$$

The energy-momentum tensors in the form of (23) and (24) for noncommutative φ^4 scalar field and noncommutative electromagnetic field have been obtained in [7, 8, 9, 10, 11] from several different methods. Here we point out that we can obtain them from a map relation between field theories on five dimensional commutative spacetime and four dimensional noncommutative spacetime. This insinuate that (23) and (24) may be the correct forms for the energy-momentum tensors of noncommutative φ^4 scalar field theory and noncommutative electromagnetic field theory.

The spectrum of time coordinate in the quantized spacetime model of Snyder is still continuous. A generalized quantum spacetime algebra with the discrete spectrum for the time coordinate has been constructed in [19]. To adopt the construction of [19], a four dimensional quantized spacetime with both its time and space coordinates discrete can be embedded in a six dimensional continuous spacetime. However in order to make the discussion of this paper simple, we will not refer to the construction of [19] in the following.

4 Energy-momentum non-conservation on noncommutative spacetime

4.1 Energy-momentum flows between four dimensional quantized spacetime and five dimensional continuous spacetime

For the energy-momentum tensors in the forms of (23) and (24) for noncommutative φ^4 scalar field and noncommutative gauge field, they do not satisfy the local conservation equation. Some modification schemes have been proposed in order to obtain a locally conserved

energy-momentum tensor for a noncommutative field theory [9, 11, 12]. However, it seems that the properties of symmetry, traceless, and gauge covariance usually cannot be satisfied together for the modified forms of the energy-momentum tensors of noncommutative field theories. In [16], a modified form for the energy-momentum tensor of noncommutative φ^4 scalar field theory has been constructed. It is symmetric and conserved. However, its expression includes path-dependent integrals. It is not the ideal form for the energy-momentum tensor of noncommutative φ^4 scalar field theory.

In this paper, we propose that energy-momentum are not locally conserved for a field theory on noncommutative spacetime. We propose that energy-momentum tensors of noncommutative field theories constructed from the Noether approach are the correct forms for the energy-momentum tensors of noncommutative field theories. In order to arrive our proposal, we look at this problem from a different angle. In section 2 we have seen that from the constructions of the four dimensional quantized spacetime \mathbf{x}^μ , its points take the eigenvalues of the operators (1). Thus its point set can be regarded as a subset of the five dimensional continuous spacetime y^μ , the four dimensional quantized spacetime \mathbf{x}^μ can be embedded in the five dimensional continuous spacetime y^μ through a certain manner, its point set composes a subspace of the five dimensional continuous spacetime y^μ . Now to observe from the five dimensional continuous spacetime y^μ where the four dimensional quantized spacetime \mathbf{x}^μ is embedded, matter and energy-momentum are existing and moving in such a five dimensional continuous spacetime, and the energy-momentum have the property of local conservation. However, there exist the exchange of matter and energy-momentum between the four dimensional sub-spacetime \mathbf{x}^μ and the five dimensional spacetime y^μ . Thus to observe from the four dimensional quantized spacetime \mathbf{x}^μ , the energy-momentum are not locally conserved necessarily. This is just like the case of a two dimensional fluid flowing in a three dimensional space. If there are no sources or holes in the two dimensional space, to observe from the two dimensional space where the fluid is flowing, there exist the continuous and conservation equations for the two dimensional fluid. However, if there exist sources and holes in the two dimensional space where the fluid is flowing, then to observe from the two dimensional space, the energy-momentum of the fluid are not locally conserved at the locations where there are sources or holes; because at these places, there exist the energy-momentum exchange (import or export) between the two dimensional space and three dimensional space. However, to observe from the three dimensional space, there still exist the continuous and conservation equations for the fluid. For the four dimensional quantized spacetime \mathbf{x}^μ embedded in a five dimensional continuous spacetime y^μ , the situation is similar. Thus there may exist the energy-momentum import from the five dimensional continuous spacetime y^μ to the four dimensional quantized spacetime \mathbf{x}^μ or the energy-momentum export from the four dimensional quantized spacetime \mathbf{x}^μ to the five dimensional continuous spacetime y^μ at every spacetime point of the four dimensional quantized spacetime.

Therefore, from the above analysis, we can understand that energy-momentum are not locally conserved necessarily to observe in the four dimensional quantized spacetime. However, to observe from the five dimensional continuous spacetime where the four dimensional quantized spacetime is embedded, energy-momentum are still locally conserved. Therefore we consider that the non-vanishing of the total divergences of the energy-momentum tensors do not mean that the energy-momentum tensors constructed from the Noether approach are not the correct forms for the energy-momentum tensors of noncommutative field theories; they just imply that there exist the energy-momentum exchange or energy-momentum

flows between the four dimensional quantized spacetime and the five dimensional continuous spacetime.

4.2 Open system on four dimensional commutative spacetime

The above analysis shows that field theories on the four dimensional quantized spacetime are not the closed systems, due to the existing of the local energy-momentum flows between the four dimensional quantized spacetime and the five dimensional continuous spacetime. They are open systems in fact. This will affect the forms of field equations. In order to elucidate the problem, we first consider an open system on the four dimensional commutative spacetime.

For the general case, we can write the energy-momentum tensor in the form

$$\mathcal{T}_{\mu\nu} = \mathcal{T}_{\nu\mu} = \begin{pmatrix} w & g_1 & g_2 & g_3 \\ S_1 & T_{11} & T_{12} & T_{13} \\ S_2 & T_{21} & T_{22} & T_{23} \\ S_3 & T_{31} & T_{32} & T_{33} \end{pmatrix}, \quad (26)$$

where w is the energy density, \mathbf{S} is the energy flux density, \mathbf{g} is the momentum density, and T_{ij} is the three-dimensional stress tensor. Supposing that there exist the adscititious local sources and flows

$$f_\mu = (e, f_1, f_2, f_3) \quad (27)$$

for the energy-momentum on the four dimensional commutative spacetime, then the conservation and motion equations take the following form

$$\partial^\mu \mathcal{T}_{\mu\nu} = f_\nu. \quad (28)$$

The four equations contained in (28) can be written down explicitly:

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathbf{S} = e, \quad (29)$$

$$\frac{\partial \mathbf{g}}{\partial t} + \nabla \cdot \mathbf{T} = \mathbf{f}, \quad (30)$$

where $\mathbf{T} = T_{ij}$ is the stress tensor. In the right hand sides of (29) and (30), (e, f_1, f_2, f_3) represent the local adscititious energy-momentum flows on the spacetime.

For the primal system, we suppose that it is composed of the fields $\varphi_r(x), r = 1, 2, \dots, s$. We suppose that the imported energy-momentum flows $f_\mu(x)$ are composed of the same matter, i.e., they are described by the same fields $\varphi_r(x), r = 1, 2, \dots, s$. We do not regard the imported energy-momentum flows $f_\mu(x)$ as the external sources. We consider that they are amalgamated with the primal system together. Thus we should use the fields $\varphi_r(x)$ to describe the whole open system. It should be correct that the local field equations

$$\frac{\partial \mathcal{L}}{\partial \varphi_r} - \frac{\partial}{\partial x_\mu} \frac{\partial \mathcal{L}}{\partial (\partial \varphi_r / \partial x_\mu)} = 0, \quad r = 1, 2, \dots, s \quad (31)$$

are still satisfied at every spacetime point for the whole open system. It is also reasonable to suppose that the expression for the local Lagrangian \mathcal{L} is not changed for such an open

system, because the field equations (31), which is just the Euler-Lagrangian equation, can be derived from the variation principle. Thus, for this open system, its Lagrangian

$$\mathcal{L} = \mathcal{L}(\varphi_r, \partial\varphi_r/\partial x_\mu) \quad (32)$$

is just the same as the Lagrangian of the closed system.

However, when the energy-momentum flows $f_\mu(x)$ are imported, the system will take an integral change. Such an integral change reflects on the integral change of the fields $\varphi_r(x)$, $r = 1, 2, \dots, s$. To view from the primal system, such an integral change for the fields is non-local; and also it violates the causality. Therefore the field theory for such an open system is a kind of non-local field theory; or it contains the component of non-local part. However, for such a non-local field system, we can still use the same local fields $\varphi_r(x)$, local Lagrangian $\mathcal{L}(\varphi_r, \partial\varphi_r/\partial x_\mu)$, and local field equations (31) to describe it; only that the local conservation and motion equations for the energy-momentum tensor are changed. They are replaced by (28) in fact. And also the non-local contents of this system are completely contained by (28). In fact, we can regard (28) as the constraint equations exerted on the system. Therefore we can adopt the method of local field theory to study such an open non-local system; but now it is a constraint system. We point out here that field theories on noncommutative spacetime can just be regarded as open non-local systems like this. Therefore such a model will be helpful for us to understand the construction of the energy-momentum tensors for field theories on noncommutative spacetime. For such a purpose, we will first make a derivation for (28)–(30) from the Noether approach.

In order to derive (28)–(30) from the Noether approach, we can suppose that under the infinitesimal displacements of the spacetime coordinates

$$x'_\mu = x_\mu + \epsilon_\mu, \quad (33)$$

the Lagrangian is changed as

$$\delta\mathcal{L} = \mathcal{L}(x') - \mathcal{L}(x) + \epsilon_\mu f^\mu = \epsilon_\mu \frac{\partial\mathcal{L}}{\partial x_\mu} + \epsilon_\mu f^\mu. \quad (34)$$

Here, the additional term $\epsilon_\mu f^\mu$ in (34) is subjected to the imported energy-momentum flows. On the other hand, from (32), because \mathcal{L} does not depend on the coordinates apparently, we have

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_r} \delta\varphi_r + \frac{\partial\mathcal{L}}{\partial(\partial\varphi_r/\partial x_\mu)} \delta\left(\frac{\partial\varphi_r}{\partial x_\mu}\right), \quad r = 1, 2, \dots, s, \quad (35)$$

where

$$\delta\varphi_r = \varphi_r(x + \epsilon) - \varphi_r(x) = \epsilon_\mu \frac{\partial\varphi_r(x)}{\partial x_\mu}. \quad (36)$$

From (34), (35), and (31), we have

$$\epsilon_\mu \frac{\partial\mathcal{L}}{\partial x_\mu} + \epsilon_\mu f^\mu = \frac{\partial}{\partial x_\mu} \left(\frac{\partial\mathcal{L}}{\partial(\partial\varphi_r/\partial x_\mu)} \epsilon_\nu \frac{\partial\varphi_r}{\partial x_\nu} \right). \quad (37)$$

Because (37) is satisfied for an arbitrary ϵ_μ , we obtain

$$\frac{\partial}{\partial x_\mu} \left(-g_{\mu\nu} \mathcal{L} + \frac{\partial\mathcal{L}}{\partial(\partial\varphi_r/\partial x_\mu)} \frac{\partial\varphi_r}{\partial x_\nu} \right) = g_{\mu\nu} f^\mu. \quad (38)$$

Then to write (38) in the form of (28), we obtain the energy-momentum tensor of the system

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \frac{\partial\mathcal{L}}{\partial(\partial\varphi_r/\partial x_\mu)} \frac{\partial\varphi_r}{\partial x_\nu} . \quad (39)$$

This means that for such an open system, its energy-momentum tensor can be expressed as the same form as that of the closed system. In fact, it is correct.

4.3 Energy-momentum tensor on noncommutative spacetime from the Noether approach

In this subsection, we give a derivation for the energy-momentum tensors of noncommutative field theories from the Noether approach. We consider the noncommutative field theories that only contain the first order derivatives in their Lagrangians written in the form of the Moyal \star -product. Thus their Lagrangians can be written as

$$\mathcal{L}_\star = \mathcal{L}_\star(\varphi_r, \partial\varphi_r/\partial x_\mu) \quad (40)$$

generally, where $r = 1, 2, \dots, s$ represent different independent field components. Although for the Lagrangians of (40), when expanded according to the parameters $\theta^{\mu\nu}$, they contain higher derivative terms, we consider that on noncommutative spacetime the Moyal \star -product is the fundamental product operation, we need not to expand the Moyal \star -product in the Lagrangians in principle. Thus field equations of $\varphi_r(x)$ can still be cast in the form of the Euler-Lagrange equation which can be derived from the variation principle. They are given by

$$\frac{\partial\mathcal{L}_\star}{\partial\varphi_r} - \frac{\partial}{\partial x_\mu} \frac{\partial\mathcal{L}_\star}{\partial(\partial\varphi_r/\partial x_\mu)} = 0 , \quad r = 1, 2, \dots, s . \quad (41)$$

For example for noncommutative φ^4 scalar field theory with the Lagrangian (15), from (41), we obtain the field equation

$$(\square + m^2)\varphi + \frac{1}{3!}\lambda\varphi \star \varphi \star \varphi = 0 . \quad (42)$$

As analyzed above, we consider that the four dimensional noncommutative spacetime is embedded in a five dimensional commutative spacetime, there exist energy-momentum flows from the five dimensional commutative spacetime to the four dimensional noncommutative spacetime. Therefore field theories on noncommutative spacetime are open systems. Like that of the open systems on the four dimensional commutative spacetime, we suppose that under the infinitesimal displacements of the spacetime coordinates

$$x'_\mu = x_\mu + \epsilon_\mu , \quad (43)$$

the displacement of the Lagrangian is given by

$$\delta\mathcal{L}_\star = \mathcal{L}_\star(x') - \mathcal{L}_\star(x) + \epsilon_\mu f^\mu = \epsilon_\mu \frac{\partial\mathcal{L}_\star}{\partial x_\mu} + \epsilon_\mu f^\mu , \quad (44)$$

where the term $\epsilon_\mu f^\mu$ is due to the existence of the energy-momentum flows from the five dimensional commutative spacetime to the four dimensional noncommutative spacetime. On the other hand, from (40), because \mathcal{L}_\star does not rely on the coordinates apparently, we have

$$\delta \mathcal{L}_\star = \frac{\partial \mathcal{L}_\star}{\partial \varphi_r} \star \delta \varphi_r + \frac{\partial \mathcal{L}_\star}{\partial (\partial \varphi_r / \partial x_\mu)} \star \delta \left(\frac{\partial \varphi_r}{\partial x_\mu} \right), \quad r = 1, 2, \dots, s, \quad (45)$$

where

$$\delta \varphi_r = \varphi_r(x + \epsilon) - \varphi_r(x) = \epsilon_\mu \frac{\partial \varphi_r(x)}{\partial x_\mu}. \quad (46)$$

From (44), (45), and (41), we have

$$\epsilon_\mu \frac{\partial \mathcal{L}_\star}{\partial x_\mu} + \epsilon_\mu f^\mu = \frac{\partial}{\partial x_\mu} \left(\frac{\partial \mathcal{L}_\star}{\partial (\partial \varphi_r / \partial x_\mu)} \star \epsilon_\nu \frac{\partial \varphi_r}{\partial x_\nu} \right). \quad (47)$$

Because (47) is satisfied for an arbitrary ϵ_μ , we obtain

$$\frac{\partial}{\partial x_\mu} \left(-g_{\mu\nu} \mathcal{L}_\star + \frac{\partial \mathcal{L}_\star}{\partial (\partial \varphi_r / \partial x_\mu)} \star \frac{\partial \varphi_r}{\partial x_\nu} \right) = g_{\mu\nu} f^\mu. \quad (48)$$

We can write (48) in the form

$$\partial^\mu \mathcal{T}_{\mu\nu} = f_\nu, \quad (49)$$

where

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu} \mathcal{L}_\star + \frac{\partial \mathcal{L}_\star}{\partial (\partial \varphi_r / \partial x_\mu)} \star \frac{\partial \varphi_r}{\partial x_\nu}. \quad (50)$$

Like that of the open system on the four dimensional commutative spacetime, we can explain (50) as the energy-momentum tensor of a field theory on noncommutative spacetime. The non-vanishing of the total divergence of the energy-momentum tensor is due to the existence of the energy-momentum flows from the five dimensional commutative spacetime to the four dimensional noncommutative spacetime.

To notice that the Moyal \star -product is not invariant generally under the commutation of the order of two functions, we need to write (50) in a symmetrized form with respect to the Moyal \star -product. Therefore we have

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu} \mathcal{L}_\star + \frac{1}{2} \left(\frac{\partial \mathcal{L}_\star}{\partial (\partial \varphi_r / \partial x_\mu)} \star \frac{\partial \varphi_r}{\partial x_\nu} + \frac{\partial \varphi_r}{\partial x_\nu} \star \frac{\partial \mathcal{L}_\star}{\partial (\partial \varphi_r / \partial x_\mu)} \right). \quad (51)$$

In fact (51) can be resulted through symmetrizing the Moyal \star -product of any two functions properly in the above derivation. For the Lagrangian (15) of the noncommutative φ^4 scalar field theory, from (51) we obtain

$$\mathcal{T}_{\mu\nu} = \frac{1}{2} (\partial_\mu \varphi(x) \star \partial_\nu \varphi(x) + \partial_\nu \varphi(x) \star \partial_\mu \varphi(x)) - g_{\mu\nu} \mathcal{L}(x), \quad (52)$$

which is just given by (23). For the Lagrangian (16) of the noncommutative electromagnetic field theory, in order to obtain a symmetric and traceless energy-momentum tensor from the

above approach, we need to invoke the Belinfante procedure at the same time. We omit to write down the detailed process here. At last we obtain

$$\mathcal{T}_{\mu\nu} = \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}(x) \star F_{\alpha\beta}(x) - \frac{1}{2}(F_{\mu\lambda}(x) \star F_{\nu}^{\lambda}(x) + F_{\nu\lambda}(x) \star F_{\mu}^{\lambda}(x)) , \quad (53)$$

which is the same form of (24). Similar procedure can be applied to the construction of the energy-momentum tensor of noncommutative $U(N)$ gauge field theory. The final expression for the energy-momentum tensor of noncommutative $U(N)$ gauge field theory is just given by (53). The expressions of (52) and (53) have been obtained previously in the literature from several different methods [7, 8, 9, 10, 11]. As pointed out in [10], the energy-momentum tensor of (53) is symmetric, traceless, and gauge covariance.

4.4 Energy-momentum non-conservation on noncommutative space-time

In fact, we have not derived the energy-momentum tensors of noncommutative field theories from the above approach really. Because we consider that the four dimensional quantized spacetime is embedded in a five dimensional continuous spacetime, there exist energy-momentum flows from the five dimensional continuous spacetime to the four dimensional quantized spacetime, energy-momentum are not locally conserved on the four dimensional quantized spacetime. In (49), we explain $f_{\mu}(x)$ as the energy-momentum flows from the five dimensional continuous spacetime to the four dimensional quantized spacetime. But here, $f_{\mu}(x)$ themselves are determined by the total divergence of the energy-momentum tensor constructed above. For such a case, $f_{\mu}(x)$ can be expressed as functions of $(\varphi_r, \partial\varphi_r/\partial x_{\mu})$, i.e.,

$$f_{\mu} = f_{\mu}(\varphi_r, \partial\varphi_r/\partial x_{\mu}) . \quad (54)$$

We explain (50) or (51) as the energy-momentum tensors of field theories on noncommutative spacetime. In addition, field theories on noncommutative spacetime are open systems, they have the property of non-locality. For closed systems on commutative spacetime, Poincaré translation invariance results the existence of locally conserved energy-momentum tensors. Thus (49) also implies that Poincaré translation invariance is violated for field theories on noncommutative spacetime.

There are some other reasons supporting us to explain (51)–(53) as the correct forms of the energy-momentum tensors of noncommutative field theories. First as pointed out in section 3, the energy-momentum tensors expressed by (51)–(53) can be obtained directly from a map relation between field theories on five dimensional commutative spacetime and four dimensional noncommutative spacetime. The second reason is that the energy-momentum tensor for a noncommutative field theory constructed from the above approach is symmetric; for gauge field theory, to combine the Belinfante mechanism together, it is symmetric, traceless, and gauge covariance. The third reason is that when we take the limit $\theta^{\mu\nu} \rightarrow 0$ in (51)–(53), which is equivalent to replace the Moyal \star -product by the ordinary product, these expressions for the energy-momentum tensors of noncommutative field theories come back to the energy-momentum tensors of corresponding field theories on commutative spacetime.

The explicit forms of $f_{\mu}(x)$ for a noncommutative field theory can be obtained from (49) and (51). For noncommutative φ^4 scalar field theory, from (52), we have [7, 8]

$$\partial^{\mu}\mathcal{T}_{\mu\nu} = \frac{\lambda}{4!} [[\varphi, \partial_{\nu}\varphi]_{\star}, \varphi^{\star 2}]_{\star} . \quad (55)$$

For the energy-momentum tensor of noncommutative gauge field theory given by (53), it satisfies the covariant conservation equation [9, 10, 11]

$$D_\mu \star \mathcal{T}^{\mu\nu} = \partial_\mu \mathcal{T}^{\mu\nu} - i(A_\mu \star \mathcal{T}^{\mu\nu} - \mathcal{T}^{\mu\nu} \star A_\mu) = 0 , \quad (56)$$

thus we have

$$\partial_\mu \mathcal{T}^{\mu\nu} = i(A_\mu \star \mathcal{T}^{\mu\nu} - \mathcal{T}^{\mu\nu} \star A_\mu) . \quad (57)$$

For the noncommutative electromagnetic field, to the first order of $\theta^{\mu\nu}$, one obtains [11]

$$\partial_\mu \mathcal{T}^{\mu\nu} = \partial_\mu [(\theta F \mathcal{T}^{(0)})^{\mu\nu} - \partial_\beta (\theta^{\alpha\beta} A_\alpha \mathcal{T}^{(0)\mu\nu})] , \quad (58)$$

where $\mathcal{T}^{(0)\mu\nu}$ is the energy-momentum tensor of electromagnetic field on commutative space-time, i.e., to let $\theta^{\mu\nu} = 0$ in (53).

To observe from the four dimensional noncommutative spacetime only, we can also explain $\partial^\mu \mathcal{T}_{\mu\nu}$ as the local generation of the energy-momentum on the four dimensional noncommutative spacetime. For the energy-momentum tensor of (51), the four-momentum over the whole three-dimensional space is given by

$$P_\mu = \int d^3x \mathcal{T}_{0\mu} . \quad (59)$$

For noncommutative φ^4 scalar field theory we have

$$E = \int d^3x \mathcal{T}_{00} = \int d^3x (\pi \star \pi - \mathcal{L}) = \int d^3x (\pi \star \dot{\varphi} - \mathcal{L}) , \quad (60)$$

$$P_i = \int d^3x \mathcal{T}_{0i} = \int d^3x \frac{1}{2} (\pi \star \partial_i \varphi + \partial_i \varphi \star \pi) , \quad (61)$$

where $\pi = \partial \mathcal{L} / \partial \dot{\varphi}$ is the conjugate momentum. The four-momentum can be written as $P^\mu = (E, P_1, P_2, P_3)$. Because

$$\int d^3x \partial^\mu \mathcal{T}_{\mu\nu} = \partial^0 \int d^3x \mathcal{T}_{0\nu} + \int d^3x \partial^i \mathcal{T}_{i\nu} = \partial^0 \int d^3x \mathcal{T}_{0\nu} , \quad (62)$$

we have

$$\partial^0 P_\mu = \int d^3x \frac{\lambda}{4!} [[\varphi, \partial_\mu \varphi]_\star, \varphi^{\star 2}]_\star . \quad (63)$$

We explain (63) as the generation of energy-momentum over the whole three-dimensional space for the noncommutative φ^4 scalar field theory. It is also necessary to point out that for the space-space noncommutativity with $\theta^{0i} = 0$, the right hand side of (63) disappears due to the integral property of the Moyal \star -product on the three-dimensional space [7]. For such a case, the energy-momentum defined by (59) for noncommutative φ^4 scalar field theory are totally conserved. However, the right hand side of (55) is still non-vanishing generally. Thus the energy-momentum of the noncommutative φ^4 scalar field are not locally conserved even if $\theta^{0i} = 0$.

We can give another argument for our proposal from the property of the propagation of nonlinear waves on noncommutative spacetime. From [20, 21], we know that nonlinear perturbations and waves of fields on noncommutative spacetime have infinite propagation

speed. From such a fact, we can understand it is possible that energy-momentum are not locally conserved on noncommutative spacetime. This is because usually local conservation of energy-momentum is a property for waves of finite propagation speed. When the propagation speed of a wave is infinite, one cannot exert the property of local conservation of energy-momentum on such a wave. For noncommutative φ^4 scalar field, we can see that the right hand side of (55) is zero for a plane wave. Thus energy-momentum are locally conserved for a plane wave of the noncommutative φ^4 scalar field. This is in accordance with the conclusion of [21] where it is pointed out that the infinite propagation speed is a property of nonlinear waves for scalar field on noncommutative spacetime. For the electromagnetic field on noncommutative spacetime, from (58) we can see that the total divergence of the energy-momentum tensor is not zero even for a plane wave. We can understand this from the non-Abelian structure (cf. (17)) of the $U(1)$ gauge field on noncommutative spacetime. Because of the existence of the non-Abelian structure, there does not exist the exact plane wave for the electromagnetic field on noncommutative spacetime in fact. Thus the waves of electromagnetic field on noncommutative spacetime are always nonlinear. They have infinite propagation speed like the nonlinear waves of scalar field on noncommutative spacetime [21]. Thus even if for a plane wave of the $U(1)$ gauge field on noncommutative spacetime, the local energy-momentum generation are not zero.

5 The existence of infinite spacetime dimension

In the previous sections, we have proposed that the four dimensional quantized spacetime \mathbf{x}^μ can be considered as a subspace embedded in a five dimensional continuous spacetime y^μ , and there exist energy-momentum flows between the four dimensional quantized spacetime \mathbf{x}^μ and the five dimensional continuous spacetime y^μ . Thus energy-momentum are not locally conserved on the four dimensional quantized spacetime \mathbf{x}^μ . However this is not the end of the problem.

Now we consider the five dimensional continuous spacetime y^μ . In such a classical and continuous spacetime, there still exist the Einstein gravitational theory and quantum theory. The principles of general relativity and quantum mechanics still take effect. According to the arguments of [2], such a spacetime cannot be a classical existence in fact. It should also be quantized under a very small microscopic scale such as the Planck scale. Thus the DFR algebra (6)–(8) should also be exerted on such a five dimensional continuous spacetime; and we will obtain a quantized five dimensional spacetime \mathbf{y}^μ at last. Field theories on such a five dimensional spacetime now also become noncommutative field theories. Thus on such a five dimensional quantized spacetime, there exist the five dimensional noncommutative scalar field theory and noncommutative electromagnetic field theory. Their Lagrangians are still given by the forms of (15) and (16); and their energy-momentum tensors are still given by the forms of (23) and (24). Similar as the case of the four dimensional noncommutative spacetime, energy-momentum are not locally conserved on such a five dimensional quantized spacetime.

On the other hand, from the constructions of quantum spacetime as analyzed in section 2, such a five dimensional quantized spacetime \mathbf{y}^μ can be embedded in a six dimensional continuous spacetime w^μ . Such a six dimensional continuous spacetime has the Lorentz group $SO(5, 1)$. In such a six dimensional continuous spacetime w^μ , there exist the commutative field theories, such as scalar field theory and electromagnetic field theory. And there exist

energy-momentum flows between the five dimensional quantized spacetime \mathbf{y}^μ and the six dimensional continuous spacetime w^μ . Thus energy-momentum are not locally conserved on the five dimensional quantized spacetime \mathbf{y}^μ .

Such an inference can be proceeded to an arbitrary spacetime dimension n . Thus we can deduce that there exists an n -dimensional quantized spacetime \mathbf{X}^μ , it is embedded in an $(n+1)$ -dimensional continuous spacetime Y^μ . However, the $(n+1)$ -dimensional continuous spacetime Y^μ also needs to be quantized according to the arguments of [2]. Therefore at last, n will tend to the limit of ∞ . Thus the total spacetime dimension is infinite. And at the same time, the infinite dimensional spacetime is quantized. Its coordinates satisfy the DFR algebra. Field theories on such an infinite dimensional spacetime are noncommutative field theories. There exist the energy-momentum flows between the ∞ -dimensional quantized spacetime \mathbf{X}^μ and the $(\infty+1)$ -dimensional continuous spacetime \mathcal{Y}^μ . However we can consider that $\infty = \infty + 1$. Therefore to observe from the whole infinite dimensional spacetime, there does not exist the local energy-momentum non-conservation. Thus the local conservation of energy-momentum for field theories on noncommutative spacetime can be realized in an ∞ -dimensional quantum spacetime. That is to say, if we demand that the energy-momentum should be locally conserved for physics, then the total dimension of the spacetime should be infinite, if the spacetime is quantized and noncommutative.

6 Some further discussions

In this paper, from the constructions of the quantum spacetime, we propose that a four dimensional quantized spacetime can be embedded in a five dimensional continuous spacetime. Thus to observe from the five dimensional continuous spacetime where the four dimensional quantized spacetime is embedded, there exist the energy-momentum flows from the five dimensional continuous spacetime to the four dimensional quantized spacetime, energy-momentum are not locally conserved on the four dimensional quantized spacetime. Thus noncommutative field systems are not closed systems; they are open systems in fact. We propose that energy-momentum tensors of noncommutative field theories constructed from the Noether approach are just the correct forms of the energy-momentum tensors of noncommutative field theories. The non-vanishing of the total divergences of the energy-momentum tensors of noncommutative field theories just reflect that energy-momentum are not locally conserved on noncommutative spacetime. At the same time, from the constructions of the quantum spacetime, we propose that the total spacetime dimension of the quantum spacetime is infinite.

However, over a long time, it is considered that energy-momentum are locally conserved and energy-momentum conservation is a fundamental principle of physics. But now, it is possible that such a conclusion may be changed in noncommutative field theories. In noncommutative spacetime, the law of energy-momentum conservation may not satisfy. As we know that noncommutative field theories have many different properties from those of field theories on commutative spacetime, such as causality, unitarity, and locality etc., we consider that energy-momentum non-conservation is another property of noncommutative field theories different from that of field theories on commutative spacetime.

From (55)–(58) for noncommutative scalar field theory and noncommutative gauge field theory, we can see that the violation of energy-momentum conservation is very weak because $\theta^{\mu\nu}$ is of order l_P^2 as postulated in the literature. Thus the effect of energy-momentum non-

conservation can only be obvious when the observed spacetime scale is comparable to the Planck scale. The energy-momentum non-conservation is a phenomenon of matter near the Planck scale where the spacetime quantization effect cannot be neglected. At the spacetime scale much larger than the Planck scale, the noncommutativity of spacetime coordinates is not obvious, the Moyal \star -product can be replaced by the ordinary product, and the physics can be described by the field theories on commutative spacetime approximately, thus energy-momentum are locally conserved approximately. Or we can say that energy-momentum conservation is an approximate phenomenon of physics in the spacetime scale much larger than the Planck scale. On the other hand, under certain circumstance and conditions, if the non-commutative parameters can be enlarged, the effect of energy-momentum non-conservation can be enlarged. In addition, we can see that in [22], from certain concrete models of non-commutative field theories, the conclusion of energy-momentum non-conservation has been derived in quantum scattering processes. However, the purpose of this paper is to propose that energy-momentum are non-conservative on noncommutative spacetime from the expressions of energy-momentum tensors of noncommutative field theories. The starting point of this paper is different from those of [22].

For the energy-momentum tensor problem of noncommutative field theories, people have tried to search for some modified forms for the energy-momentum tensors of noncommutative field theories. However, it seems that the solutions are not satisfied. Energy-momentum conservation is a conviction of physics. It seems that such a conception is related with a recognition on the matter of people, which is that the matter have the absolute objective reality. If the matter have the absolute objective reality, it is reasonable that energy-momentum should possess the property of conservation. However, whether the matter have the absolute objective reality really seems doubtful. This is because when the spacetime is quantized, the classical meaning of the spacetime does not exist any longer. Spacetime coordinates have become noncommutative operators. Or we can say that the traditional concept of the spacetime depending on the experience does not exist when the spacetime is quantized. Thus the spacetime does not have the absolute objective reality when it is quantized. It is difficult to imagine that the matter whose existence relies on the existence of spacetime can possess the absolute objective reality when the spacetime itself does not possess the absolute objective reality. Thus we consider that the matter whose existence relies on the existence of spacetime do not have the absolute objective reality when the spacetime is quantized. We consider that the conception of energy-momentum conservation is related with the conception that the matter are absolute objective reality. When the conception that the matter are absolute objective reality is doubtful, the conception of energy-momentum conservation will also be doubtful. Therefore we propose that energy-momentum are non-conservative on noncommutative spacetime; the energy-momentum tensors of noncommutative field theories constructed from the Noether approach in subsection 4.3 are just the correct forms for the energy-momentum tensors of noncommutative field theories.

In section 5, from the constructions of quantum spacetime and spacetime quantization principle, we propose that the total spacetime dimension is infinite, and it is quantized. Under the meaning that the matter are existing and moving in an infinite dimensional spacetime, the local conservation of energy-momentum can be realized. For such an infinite dimensional spacetime, it seems that there is no reason to assume that it is compactified for its dimensions other than the first four. From the analysis of this paper, we can see that there can at least exist the connections between the matter in the observed four dimensional spacetime and

the additional higher dimensions through the exchange of the energy-momentum between them.

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